

The scientific achievements of the project are the following:

- *The introducing of equality algebras (Studia Logica).*
A new structure, called equality algebras, is introduced. It has two connectives, a meet operation and equivalence, and a constant. A closure operator is defined in the class of equality algebras, and we call the closed algebras equivalential. We show that equivalential equality algebras are term equivalent with BCK-algebras with meet. As a by-product, we obtain a quite general result, which is analogous to a result of Kabziński and Wroński (on Heyting-algebras and Intuitionistic Logic): we provide an equational characterization for the equivalential fragment of BCK-algebras with meet. Residuated lattices, being the algebraic semantics for substructural logics, have gained considerable importance [N. Galatos, P. Jipsen, T. Kowalski, H. Ono, Residuated Lattices: An Algebraic Glimpse at Substructural Logics, Volume 151. Studies in Logic and the Foundations of Mathematics (2007) pp. 532]. BCK-algebras with meet are exactly the $(\wedge, \rightarrow, 1)$ -subreducts of residuated lattices. Therefore, our result may serve as a precursor for an axiomatization of substructural logics based on equivalence rather than on implication, in accordance with the original intention of Leibniz for Classical Logic.
- *Pseudo Equality Algebras (the paper is under review)*
A generalization of equality algebras to the non-commutative setting has been written and submitted: [S. Jenei, L. Kóródi, Pseudo Equality Algebras]. The paper also contains results on the variety of equality algebras and of pseudo equality algebras. The paper is under review.
- *An investigation of the link between the rotation-construction and Abelian groups (Fuzzy Sets and Systems).*
Uninorm Logic and its schematic extensions have their important place in Mathematical Fuzzy Logic as described in [G. Metcalfe, F. Montagna, Substructural Fuzzy Logics, J. Symbolic Logic, 72, Issue 3 (2007), 834–864.]. I introduced a new construction in the paper entitled "On the relationship between the rotation construction and ordered Abelian groups", and gave a characterization for the class of uninorms that can be constructed with it.
By analogy with the usual extension of the group operation from the positive cone of an ordered Abelian group into the whole group, a construction—called symmetrization—is defined and it is related to the rotation construction [Jenei, On the structure of rotation-invariant semigroups, Archive for Mathematical Logic 42 (2003) 489–514]. Symmetrization turns out to be a kind of dualized rotation. A characterization is given for the left-continuous t-conorms for which their symmetrization is a uninorm. As a by-product a new family of involutive uninorms is introduced.
- *A geometric flavor study of residuated lattices (Annals of Pure and Applied Logic).*

The main meta-mathematical outcome of the paper entitled "On the reflection invariance of residuated chains" is to point out that the structural description of residuated structures requires the usage of the co-residuated setting. This finding is quite surprising taking into account that the mathematical investigation of residuated structures has been started 70 years ago on one hand, and this seems to be the first observations in this direction on the other.

It is shown that, under certain conditions, a subset of the graph of a commutative residuated chain is invariant under a geometric reflection. This result implies that a certain part of the graph of the monoidal operation of a commutative residuated chain determines another part of the graph via the reflection on one hand, and tells us about the structure of continuity points of the monoidal operation on the other. Finally, these results are applied for the subdomains of uniqueness problem, yielding new results.

- *Structural description of a class of involutive uninorms via introducing skew symmetrization (J Logic and Computation).*

I gave a structural description for a class of involutive uninorm chains. An unusual generalization of the well-known cone-representation of ordered groups is introduced and it has been proven that certain involutive uninorm chains can be represented in this way. This result may serve to be the precursor to addressing the proof of standard completeness of IUL logic, a problem that has been left open in [G. Metcalfe, F. Montagna, Substructural Fuzzy Logics, J. Symbolic Logic, 72, Issue 3 (2007), 834–864]. The main ‘philosophical’ outcome of this paper is to demonstrate that the structural description of residuated lattices requires the use of the co-residuated setting. A construction, called skew symmetrization, which generalizes the well-known representation of an ordered Abelian group obtained from the positive (or negative) cone of the algebra is introduced here. Its definition requires leaving the accustomed residuated setting and entering the co-residuated setting. It is shown that every uninorm on $[0,1]$ with an involution defined by the residual complement with respect to the unit and having the unit as the fixed point of the involution can be described as the skew symmetrization of its underlying t-norm or underlying t-conorm.

- *Algebraic investigation of involutive FLe-monoids (Archive for Mathematical Logic).*

The monoidal operation of involutive FLe-algebras is a commutative, residuated monoid with an involutivity condition; call it an involutive uninorm. Our main question is the following: in an involutive FLe-algebra, how far its uninorm (or its algebraic structure, in general) is determined by its “local behavior”, i.e., its underlying t-norm and t-conorm. An answer for a particular case on $[0,1]$ has already been presented in [S. Jenei, Structural description of a class of involutive uninorms via skew symmetrization]; see the previous item. It says that the uninorm is determined uniquely by any of them, i.e., either by the t-norm or by the t-conorm. In fact, the t-norm and the t-conorm are determined by each

other, in this case. Then, a natural question is how far we can extend this, and when the uninorm is determined uniquely? The paper deals with involutive FLe-monoids, that is, commutative residuated, partially-ordered monoids with an involutive negation. Involutive FLe-monoids over lattices are exactly involutive FLe-algebras, the algebraic counterparts of the substructural logic IUL. A cone representation is given for conic involutive FLe-monoids, along with a new construction method, called twin-rotation. Some classes of finite involutive FLe-chains are classified by using the notion of rank of involutive FLe-chains, and a kind of duality is developed between positive and non-positive rank algebras. As a side effect, it is shown that the substructural logic IUL plus $t \leftrightarrow f$ does not have the finite model property.

- *The introducing and classifying of strongly involutive uninorm algebras along with complexity issues of the related logic (J Logic and Computation).*
We investigate uninorm algebras satisfying a strong version of involutiveness. More precisely, we require that negation is an order reversing monoid isomorphism between the positive cone and the negative cone. A rather surprising consequence of this property is that the negative cones of these algebras are BL-algebras, which do not admit MV-components with more than two elements. Among other things, we prove standard completeness and co-NP completeness of the logic corresponding to these algebras.

- *Classification of absorbent-continuous sharp FLe-algebras over subreal chains (the paper is under review)*
The classification of a certain kind of involutive FLe-chains is given: We define absorbent-continuity and sharpness of involutive FLe-algebras, along with the notion of subreal chains, and classify absorbent-continuous, sharp FLe-algebras over subreal chains: The algebra is determined by its negative cone, and the negative cone can only be chosen from a certain subclass of BL-chains, namely, one with components which are either cancellative (that is, those components are negative cones of totally ordered Abelian groups) or two-element MV-algebras, and with no two consecutive cancellative components. It is shown that the classification theorem does not hold if we drop the absorbent continuity condition. Our result is the first classification theorem in the literature that doesn't assume the naturally ordered condition. By replacing it by the much weaker absorbent continuity one can still obtain a classification.

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